Math 2150 - Homework # 1

What is a differential equation?

1. For the following differential equations, determine (i) if it is a partial differential equation (PDE) or an ordinary differential equation (ODE) and (ii) what it's order is. If it is an ODE then also determine if it is (iii) linear or non-linear.

   (a) $y' = e^{2x} + \sin(x)$

   (b) $y'' - 4y = 0$

   (c) $y''' + 3y'' + 4y' + 12y = x^2 + x - 1$

   (d) $x^2y''' - 5y'' + \sin(x)y' - 2y = \cos(x) - 2$

   (e) $\dfrac{d^2y}{dx^2} + yx^3\dfrac{dy}{dx} + x^2y = 0$

   (f) $\sin(x^2)y' + y = x$

   (g) $(2xy - y^3) + e^x\dfrac{dy}{dx} = 0$

   (h) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial^2 u}{\partial t^2}$

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2. Let $f_1(x) = e^{2x}$ and $f_2(x) = e^{-2x}$.

   (a) Show that f_1 and f_2 are both solutions to the differential equation

   $$y'' - 4y = 0$$

   on the interval $I = (-\infty, \infty)$.

   (b) Show that f_1 is a solution to the initial value problem

   $$y'' - 4y = 0, \quad y'(0) = 2, \quad y(0) = 1$$

(c) Show that f_2 is a solution to the initial value problem

$$y'' - 4y = 0, \;\; y'(1) = -2e^{-2}, \;\; y(1) = e^{-2}$$

(d) Show that if c_1 and c_2 are any constants that

$$f(x) = c_1 f_1(x) + c_2 f_2(x) = c_1 e^{2x} + c_2 e^{-2x}$$

satisfies

$$y'' - 4y = 0$$

on the interval $I = (-\infty, \infty)$.

(e) Find constants c_1 and c_2 so that

$$f(x) = c_1 e^{2x} + c_2 e^{-2x}$$

satisfies

$$y'' - 4y = 0, \;\; y'(0) = 0, \;\; y(0) = 1$$

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3. Let $\phi(x) = 2\sqrt{x} - \sqrt{x}\ln(x)$.

(a) Show that ϕ is a solution to the differential equation

$$4x^2 y'' + y = 0$$

on the interval $I = (0, \infty)$.

(b) Conclude that ϕ is a solution to the initial value problem

$$4x^2 y'' + y = 0, \;\; y'(1) = 0, \;\; y(1) = 2$$

on the interval $I = (0, \infty)$.

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4. Show that $f_1(x) = e^{-3x}$ and $f_2(x) = \cos(2x)$ and $f_3(x) = \sin(2x)$ are each solutions to the differential equation

$$y''' + 3y'' + 4y' + 12y = 0$$

on the interval $I = (-\infty, \infty)$.
